Tribhuvan University
Institute of Science and Technology
Bachelor of Science in Computer Science and Information Technology
Detailed Syllabus

Course Title: Discrete Structure (CSC 152) **Full Marks: 80+20**
Credit hours: 3 **Pass Marks: 32+8**
Nature of Course: Theory (3 hours)

Course Synopsis: This course contains the fundamental concepts of logic, reasoning and algorithms.

Goal: After completing this course, the target student will gain knowledge in discrete mathematics and finite state automata in an algorithmic approach. It helps the target student in gaining fundamental and conceptual clarity in the area of logic, reasoning, algorithms, recurrence relation, and graph theory.

Course Contents:

Unit 1: Logic, induction and Reasoning **(12 hrs.)**

1.1. Logic

Propositions, truth table, propositional logic, compound propositions, logical connectives (negation, conjunction, disjunction, exclusive OR, implication, converse, contrapositive and biconditional, truth table for each), translation of English statements into logical expressions, application of logic in Boolean searches.

Contradiction, tautology, contingency, logical equivalences, laws of logical equivalences (identity, domination, laws, double negation, commutative, associative, distributive, absorption and DeMorgan's), proving logical equivalence by using truth tables and laws of logical equivalences, dual of compound proposition.

Predicate logic, quantifiers and quantification, universal and existential quantification, translation of quantified expressions into English sentences and vice-versa, bounded and free variables, negations of quantified expression

Examples related to each concept.

1.2 Induction and Reasoning

Rules of inferences (addition, simplification, conjunction, modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and resolution), proving the validity of arguments, resolution, fallacies (affirming and conclusion, denying the hypothesis and circular reasoning), quantified statements, rules of inference for quantified statements

(universal instantiation, universal generalization, existential instantiation and existential generalization)

Methods of proving theorems (trivial proof, vacuous proof. Direct proof, indirect proof, proof by contradiction, proof by cases, existence proof, proof by counter example, proof by weak and strong inductions, recursive induction)

Examples related to each concept.

Unit 2: Finite State Automata **(10 hrs)**

2.1 Deterministic Finite Automata

Concept of sequential circuit design, use of Finite State Machine in sequential circuit design, Finite State Machine with output and without output, formal definition of Deterministic Finite Automata, concatenation, kleene closure, application of DFA, language of DFA, examples of languages accepted by DFA, related problems.

2.2 Non-Deterministic Finite Automata

Formal definition of non-deterministic finite automata, difference between NFA and DFA, language of NFA, examples of languages accepted by NFA

Related problems in each case

2.3 Regular Expressions

Formal definition of regular expressions, regular languages, similarities in regular expressions and automatons, application of regular expressions, related exercises.

2.4 Languages and Grammars

Natural language and formal language, syntax and semantics, phrase structure grammar, derivation, Language generated by grammar, Types of phrase structure grammar, Related problems

Unit 3: Recurrence Relations **(8 hrs.)**

3.1 Recurrence Relation

Introduction to recurrence relations, definition of recurrence relations, Fibonacci numbers, recursively defined sets and structures, basic concepts of combinatory (sum and product rules, the Pigeonhole principle, permutation, combination, binomial coefficients), initial condition, modeling problems with recurrence relation, related exercises.

3.2 Solution of Recurrence Relation

Definition and examples of linear homogeneous recurrence relation, Verification of following theorems without proof:

Theorem 1 (page 414) : Let c_1 and c_2 be two real number. Suppose $r^2 – c_1r – c_2 = 0$ has two distinct roots r_1 and r_2. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2}$ if and only if $a_n = \alpha_1r_1^n + \alpha_2r_2^n$ for n = 0, 1, 2, … where α_1 and α_2 are constants.

Theorem 2 (page 416) : Let c_1 and c_2 be two real numbers with $c_2 \neq 0$. Suppose $r^2 – c_1r – c_2 = 0$ has only one root r_0. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1a_{n-1} + c_2 a_{n-2}$ if and only if $a_n = \alpha_1r_0^n + \alpha_2nr_0^n$, for $n = 0, 1, 2, \ldots$, where α_1 and α_2 are constants.

Theorem 3 (page 417): Let c_1 and c_2 …… c_k be k real numbers. Suppose that the characteristic equation $r^k- c_1r^{k-1}\ldots\ldots – c_k = 0$ has k distinct roots $r_1, r_2,\ldots r_k$. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots. c_ka_{n-k}$ if and only if $a_n = \alpha_1r_1^n + \alpha_2r_2^n \ldots\ldots + \alpha_kr_k^n$, for n = $0, 1, 2, \ldots$, where $\alpha_1, \alpha_2, \ldots\ldots \alpha_k$ are constants.

Theorem 4 (page 418): Let c_1 and c_2 ……… c_k be k real numbers. Suppose that the characteristic equation $r^k- c_1r^{k-1} \ldots– c_k = 0$ has t distinct roots $r_1, r_2, \ldots r_t$ with multiplicities $m_1, m_2, m_3 \ldots\ldots\ldots m_t$, respectively Then a sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots. c_ka_{n-k}$ if and only if $a_n = (\alpha_{1,0} + \alpha_{1,1}\, n + \ldots + \alpha_{1,m1-1}\, n^{m1-1})\, r_1^n +(\alpha_{2,0} + \alpha_{2,1}\, n + \ldots\ldots + \alpha_{2,m2-1}\, n^{m2-1})\, r_2^n + \ldots + \alpha_{t,0} + \alpha_{t,1}\, n + \ldots + \alpha_{t,mt-1}\, n^{mt-1})\, r_t^n + for\ n = 0, 1, 2,\ldots$, where $\alpha_{i,j}$ are constants for $1 \le i \le t$ and $0 \le j \le m_i -1$

Definition and example of linear nonhomogeneous, associated homogeneous recurrence relations, verification of following theorems without proof:

Theorem 5 (page 420): If $\{a_n^{(p)}\}$ is a particular solution of the nonhomogeneous recurrence relation with constant coefficients $a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots c_ka_{n-k} + F(n)$, then every solution of the form $\{a_n^{(p)} + a_n^{(h)}\}$, where $\{a_n^{(h)}\}$ is solution of associated homogeneous recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots\ldots\ldots c_ka_{n-k}$

Theorem 6 (page 421): Suppose that $\{a_n\}$ satisfies the linear homogeneous recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots\ldots + c_ka_{n-k} + F(n)$, where $c_1\ c_2 \ldots\ldots c_k$ are real numbers and $F(n)=(b_tn^t + b_{t-1}n^{t-1} + \ldots+ b_1n + b_0)s^n$,where $b_0, b_1, \ldots b_t$ and s are real numbers. When s is not root of characteristic equation of the associated linear homogeneous recurrence relation, there is a particular solution of the form $(p_tn^t + p_{t-1}n^{t-1} + \ldots.+ p_1n + p_0)s^n$, where s is root of this characteristic equation and it's multiplicity is m, there is a particular solution of the form $n^m(p_tn^t + p_{t-1}n^{t-1} + \ldots.+ p_1n + p0)s^n$.

Solution of recurrence relation to algorithm analysis, divide and conquer relations, partial order relation. Examples illustrating each

Unit 4: Graph Theory (15hrs)

4.1 Introduction

Definition of directed and undirected graphs, simple and multigraphs, pseudographs, edges, vertices, isolated and pendant vertices, adjacent vertices, incident edge, adjacent and incident matrices representations of graphs, adjacency lists, in-degree, out-degree of a vertex, **regular graph**, Graph isomorphism.

Statement of *Handshaking Theorem* (Theorem 1, page 546), proof of Theorem 2, page 547, statement of Theorem 3, page 548, verifications of the theorems by examples.

Examples of some special graphs (**round-robin tournaments**, the complete graph K_n on n vertices, the cycles C_n, the wheel W_n, n-cube Q_n), bipartite graphs and complete bipartite graphs $K_{m,n}$, **application of graphs on local-area networks**, subgraph of a graph and union of graphs. Examples related to all concepts.

4.2 Graph Connectivity

Definitions and examples of walk and path and circuits, cut vertices/Edges, cut sets, connectedness in undirected and directed graphs, weakly and strongly connected graphs, underlying graphs, connected components. Proof of Theorem 1, page 570 (there is a simple path between every pair of distinct vertices of a connected undirected graph), related exercises.

4.3 Euler and Hamiltonian Graphs

Definitions and examples of Euler paths and circuits, multigraph model of the town of Koenigsberg, proof of Theorem 1, page 581 (A connected autograph has an Euler circuit if and only if each of it's vertices has even degree), Algorithm 1 (page 581) *Constructing Euler Circuits*, example illustrating the algorithm, proofs of Theorem 2, page 582 (a connected multigraph has an Euler path but not an Euler circuit if and only if exactly two of it's vertices has odd degree), related exercises.

Definitions of Hamiltonian paths and circuits, examples illustrating existence and nonexistence of Hamiltonian circuit, K_n has Hamiltonian circuit whenever $n>2$, application of *Dirac's Theorem* and *Ore's Theorem*, application of Hamiltonian circuit to travelling salesman problem, related exercises.

4.4 The Shortest Path Algorithm

Definition of weighted graph, the *Shortest Path Algorithm of Dijkstra*, examples to illustrate the algorithm, idea of traveling salesman problem in connection to the shortest path algorithm, computational difficulty of the traveling salesman problem, related exercises.

4.5 Planar Graphs

Definition and examples of planar graphs, statements of the following results without proof:

Theorem 1 (page 606). Let G be a connected planner simple graph with e edges and v vertices. Let r be the number of regions in a planer representation of G, then $r = e - v + 2$.

Corollary 1 If G is connected planer simple graph with e edges and v vertices where $v \geq 3$, then $e \leq 3v - 6$.

Corollary 2 If *G* has a connected planar simple graph, then it has a vertex of degree not exceeding five.

Corollary 3 If G is connected planer simple graph with e edges and v vertices where $v \geq 3$ and no circuits of length 3, then $e \leq 2v - 4$.

Homeomorphism, Related examples illustrating the results in every cases.

4.6 Graph Coloring

Basic concept, definition of Chromatic number, the statement of *Four Color Theorem* (The chromatic number of a graph is no greater than four), applications of graph colorings to scheduling final exams, illustrating the concepts by examples.

4.7 Trees and Spanning Trees

Definitions and examples of tree and forest, parent, child, rooted tree, ordered rooted tree ancestor, descendent, sibling, level/depth of node, depth of a tree, internal nodes, leaf nodes, subtree, binary tree. Definitions of *m-ary* and full *m-ary* tree, trees as models (computer file systems, tree-connected parallel-processors), related examples.

Proofs of Theorem 1**,** page 633 (An undirected graph is tree if and only if there is a unique simple path between any two of it's vertices) and Theorem 2**,** page 638 (A tree with n vertices has $n-1$ edges). Statement of Theorem 3**,** page 639, (A full *m-ary* tree with i internal vertices contains $n=mi+1$ vertices). Verifications of the results by examples.

Applications of trees (Algorithm 1, page 646, *Binary Search Tree Algorithm*), decision trees, Prefix codes, related examples.

Definition of spanning tree, proof of Theorem 1, page 676 **(**a simple graph is connected if and only it has spanning tree), finding minimal spanning tree by using Kruskal's algorithm, related exercises.

4.8 Network Flow Problems

Concept of network flows, proof of Maxflow and Mincut theorem, verification of the algorithms by examples.

Text Books:

1. Kenneth H. Rosen, *Discrete Mathematical Structures with Application to Computer Science*, WCB/McGraw Hill.

2. Joe L. Mott, Abrahan Kandel and Theodore P. Baker, *Discrete Mathematics for Computer Scientists and Mathematicians*, Prentice-Hall of India.

Reference Books:

1. R. Johnsonbaugh, *Discrete Mathematics*, Prentical Hall Inc.
2. G. Chartand, B.R. Oller Mann, *Applied and Algorithmic Graph Theory*, McGraw Hill.
3. G. Birkhoff, T.C. Bartee, *Modern Applied Algebra*, CBS Publishers.

Remarks:

1. Theory and practice should be done side by side.
2. Theory classes **4hrs** and tutorial classes **2 hrs** per week.
3. Recommended to use Mathematica/Matlab/Maple for testing selected exercises.

Subject Expert:
Dr. Tanka Nath Dhamala
Participants:
1. Arjun Singh Saud
2. Dinesh Khadka
3. Laxmi Rayamajhi (Rawal)
4. Jay Narayan Jha
5. Manoj Kumar Gupta
6. Nawaraj Paudel

Marks distribution:
1. Unit 1: 24 marks (2 in group A, 1 in group B, and 2 in group C)
2. Unit 2: 8 marks (2 in group A and 1 in group B)
3. Unit 3: 16 marks (2 in group A, 1 in group B, and 1 in group C)
4. Unit 4: 32 marks(4 in group A, 2 in group B, and 2 in group C)